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Nijman, T.E.; Verbeek, M.J.C.M.; van Soest, A.H.O.

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UNIVERSITEIT BRABANT

POSTBOX 90153
5000 LE TILBURG
THE NETHERLANDS

[redacted]

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RESEARCH MEMORANDUM

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Theo Nijman
Marno Verbeek
Arthur van Soest

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THE OPTIMAL DESIGN OF ROTATING PANELS IN A SIMPLE ANALYSIS OF VARIANCE MODEL

Theo Nijman*)
Marno Verbeek*)
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Abstract

In this paper we consider the estimation of linear combinations of period means from rotating panels in a simple analysis of variance model. Results from spectral theory are used to obtain manageable expressions for an efficient estimator of the parameters and its variance. Conditions on the relative sample size are derived under which one rotation scheme will yield more efficient estimators than another of the period means themselves, of differences or of averages of means. Moreover we present conditions under which one rotation scheme will be preferable to another irrespective of the parameter of interest. Results are presented for the case in which one is interested in estimates of the means in recent periods as well as the case of estimation of means in a more distinct past.
Our analysis shows that the gains from choosing an optimal rotation design can be quite substantial, even in case the cost of a repeated observation on the same individual equals the cost of a first observation. In many cases either the smallest or the highest possible rotation period is optimal. In a numerical example concerning monthly consumer expenditures on food, a rotating panel with a rotation period of 4 months will yield an efficiency gain of over 70 % if one is estimating a difference in subsequence consumption levels, compared to a series of independent cross sections with the same number of observations in each month.

*) Department of Econometrics, Tilburg University, P.O.Box 90153, 5000 LE Tilburg, The Netherlands. We thank Rob Alessie and Gerard van den Berg for helpful comments. Financial support by the Netherlands Organisation for Scientific Research (N.W.O.) is gratefully acknowledged by the second author.

1. Introduction

Consider the choice which a data collecting agency will have to make in order to monitor e.g. average expenditures on some consumption category either to interview the same individual in several periods or to interview different individuals in different periods. It is well known in the literature that the optimal design of the data will in general depend on the parameter to be estimated (see e.g. Raj [1968, p. 152 ff.] or Cochran [1977, p. 345 ff.]). Little attention however seems to have been paid to the optimal design of rotating panels. In the early literature, Patterson [1950] and Eckler [1955] pay attention to the estimation of a time dependent mean from several kinds of rotating samples, i.e. samples with partial replacement of units, and to the resulting variances. Biørn [1981] discusses estimation from a rotating panel in which exactly one half of the individuals is replaced every period, but does not pay attention to the question why this or any other rotation scheme should be preferable.

In this paper we concentrate on the estimation of linear combinations $\Sigma_{j=0}^{J}\xi_j\mu_{-j}$ of the period means μ_t in the simple analysis of variance model

$$y_{it} = \mu_t + \alpha_i + \varepsilon_{it} \quad (i = 1,2,\ldots,N;\ t = -T,\ -T+1,\ldots,0,\ldots,S) \qquad (1)$$

where the α_i and ε_{it} are i.i.d. normal random variables with mean zero and variances σ_α^2 and σ_ε^2 respectively which are mutually independent and independent of the unknown constants μ_t. Throughout this paper we assume for simplicity that the parameters σ_α^2 and σ_ε^2 are known a priori. If these parameters are unknown and replaced by consistent estimates the same results hold true asymptotically. The constant correlation over time between different observations on the same individual implied by (1) is considered for analytical convenience only. The analysis can easily be extended to more general correlation patterns. Moreover, if no unit is observed for more than two periods (1) is not restrictive.

In the first four sections we restrict ourselves to the estimation of period means not too close to the beginning or end of the period over which observations are available, because we will present results for the limiting case when T and S tend to infinity only. In Section 5 we drop the assumption that an infinite number of future observations is available at the time of estimation. Instead we assume S to be fixed. As a special case we consider the case in which no future observations are available, i.e. S = 0.

Define a rotating sample with rotation length r by the property that in every period 100 r^{-1} % of the participants is replaced and assume that those units are replaced which already participate the largest number of periods. If e.g. r = 2, 50% of the participants in the first wave of the rotating sample will be replaced in the second wave, the other half is replaced in the third wave. New participants in the second wave will be replaced in the fourth wave etc. Of course a rotating sample with rotation period equal to one is simply a series of cross sections. Note that this concept of rotating panels, apart from the sampling structure in the first and final r-1 periods, corresponds with the concept of 'multi-level rotation sampling' given by Eckler [1955]. In this paper we analyse the question when a rotating sample with rotation length r and n_r observations in every wave will be more informative on some linear combination of the period means μ_t in (1) than another sample with rotation length s and n_s observations in every wave.

The plan of this paper is as follows. In Section 2 we will show how results from spectral analysis can be used to obtain manageable expressions for the variance of efficient estimators of linear combinations of the μ_t in (1). In Section 3 these results are used to derive conditions under which one design will be preferable to another for some selected linear combinations of interest. Defining $\rho = \sigma_\alpha^2(\sigma_\varepsilon^2+\sigma_\alpha^2)^{-1}$, we show that if one is e.g. interested in precise estimation of the period means themselves only, a cross section will be more informative than a rotating panel with r = 2 if $n_1 > (1-\rho^2)^{-1/2}\, n_2$ and the rotating sample will be more informative if this condition does not hold. In Section 4 conditions are derived under which one design will unambiguously be preferable to another. We show that if $n_1 > (1-\rho)^{-1}\, n_2$ a cross section will yield more efficient estimates of *any* linear combination of the period means than a rotating panel with rotation period $r = 2$, while the opposite is true if $n_1 < (1+\rho)^{-1} n_2$. In Section 5 the results in Section 3 are extended and bounds are derived for specific parameters of interest assuming that y_{it} is observed if $t = -T,..,S$ for some fixed S. Section 6 concludes.

2. Theoretical results on the variances of the parameter estimators

In this section we will discuss the main steps in the derivation of manageable expressions for the variance of efficient estimators of linear combinations of the period means in (1) from a rotating sample with rotation period r. Details are presented in the appendix.

Define the vectors $\mu' = (\mu_{-T},\dots,\mu_S)$ and $\xi' = (\xi_S,\dots,\xi_{-T})$ such that $\xi'\mu = \Sigma_{j=0}^{J}\xi_j\mu_{-j}$. Using the fact that the data in a rotating panel with rotation period r can be divided in r independent subsamples in such a way that every subsample is a time series of independent small panels, we first show in the appendix that

$$\xi'\hat{\mu} \sim N(\xi'\mu, \frac{\sigma_\varepsilon^2}{n_r}\xi'V\xi) \tag{2}$$

where $\hat{\mu}$ is the efficient estimator of μ from the rotating panel under consideration and V is defined by $V = A^{-1}$, where A is a band matrix satisfying $A_{\ell k} = a_{|\ell-k|}$ if $r-T < \ell,k < S-r$ and

$$\begin{aligned} a_\tau &= 1 - \frac{\rho}{1+(r-1)\rho} && \text{if } \tau = 0 \\ &= -\frac{r-\tau}{r}\,\frac{\rho}{1+(r-1)\rho} && \text{if } 0 < \tau < r \\ &= 0 && \text{if } \tau \geq r. \end{aligned} \tag{3}$$

The main problem then is to find expressions for the elements of A^{-1}. A similar problem has been analysed in the literature on time series analysis, where the inversion is considered of a matrix Σ^{MA} defined by $\Sigma_{ts}^{MA} = E\, x_t x_s$, with x_t generated by some moving average process, $x_t = \vartheta(L)e_t$ where $e_t \sim NID(0,\sigma_e^2)$ and $\vartheta(L) = 1 + \vartheta_1 L + \ldots + \vartheta_{r-1}L^{r-1}$ is a polynomial in the lag operator L. It is well known that the inverse of Σ^{MA} can be approximated by the matrix Σ^{AR} defined by $\Sigma^{AR}_{ts} = Ez_t z_s$ where z_t is the autoregressive process obtained by inverting the lag polynomial in the moving average process underlying Σ^{MA}, that is $z_t = \vartheta^{-1}(L)e_t$. More precisely, Shaman [1975] shows that Σ^{MA} and $(\Sigma^{AR})^{-1}$ are identical except for the $(r-1)\times(r-1)$ submatrices in the upper left and lower right corners.

Now choose ϑ_k $(k=0,\ldots,r-1)$ in such a way that $Ex_t x_s = a_{|t-s|}$ which is possible because the a_τ $(\tau=0,\ldots,r-1)$ satisfy the conditions given by Wold [1953, pp. 152-154]. If Σ^{MA} and Σ^{AR} are chosen in this way the matrix A which is to be inverted differs from Σ^{MA} only by $(r-1)\times(r-1)$ submatrices in the upper left and lower right corners. Using this fact and the result obtained by Shaman [1975] we show in the appendix that if $\xi_j = 0$ for $|j| > J$ then

$$\lim_{S,T\to\infty} \xi' A^{-1} \xi = \lim_{S,T\to\infty} \xi' \Sigma^{AR} \xi. \tag{4}$$

As $A^{-1} = V$ equation (4) shows how the variance of a linear combination of period means not too close to the beginning or end of the observation period can be computed.

The simplest way to obtain the elements of the matrix Σ^{AR} (and to obtain the results to be presented in Section 4) appears to be to use results from spectral analysis. The spectral density associated with the series of covariances a_τ is defined by

$$f(\lambda) = \frac{1}{2\pi} \sum_{\tau=-r+1}^{r-1} a_{|\tau|} e^{-i\lambda\tau} \qquad , -\pi < \lambda < \pi . \tag{5}$$

In the appendix we show that if the a_τ are generated by (3) $f(\lambda)$ can be written as

$$f(\lambda) = \frac{1}{2\pi} \frac{1}{1+(r-1)\rho} \{ 1 - \rho + \rho r - \frac{\rho}{r} \frac{1 - \cos(\lambda r)}{1 - \cos\lambda} \}, \qquad \lambda \neq 0$$
$$= \frac{1}{2\pi} \frac{1}{1+(r-1)\rho} \{ 1 - \rho \}, \qquad \lambda = 0. \tag{6}$$

A direct consequence of standard results in spectral analysis, see e.g. Fishman [1969], Priestley [1981] or Harvey [1981], is the fact that the variance of $\sum_{j=0}^{J} \xi_j z_{t-j}$, where $z_t = \theta^{-1}(L)e_t$ as before, can be written as

$$\mathrm{Var}(\sum_{j=0}^{J} \xi_j z_{t-j}) = \frac{1}{4\pi^2} \int_{-\pi}^{\pi} g(\lambda)\ f^{-1}(\lambda)\ d\lambda \tag{7}$$

where $f^{-1}(\lambda)$ denotes $1/f(\lambda)$ and

$$g(\lambda) = \sum_{j=-J}^{J} w_{|j|}\ e^{i\lambda j} \tag{8}$$

with

$$w_k = \sum_{j=0}^{J} \xi_j \xi_{j+k} \qquad (k=0,\ldots,J). \tag{9}$$

As $\mathrm{Var}(\sum_{j=0}^{J} \xi_j z_{t-j})$ can also be written as $\xi' \Sigma^{AR} \xi$ we finally obtain the main result of this section from (4) and (7):

$$\xi'\hat{\mu} \sim N(\xi'\mu,\ \frac{\sigma_\varepsilon^2}{4 n_r \pi^2} \int_{-\pi}^{\pi} g(\lambda)\ f^{-1}(\lambda)\ d\lambda\ ) \tag{10}$$

provided $\xi_j = 0$ if $|j| > J$ for some finite J.

3. The optimal choice of the rotation period for specific parameters of interest

Result (10) in the previous section shows how the variance of an efficient estimator will depend on the linear combination of the period means to be estimated, on the choice of the rotation period and on the number of observations in each wave. In this section we will analyse what this result implies for the optimal choice of the rotation period r if one is interested in some particular linear combination of the period means in (1). In the next section we will use (10) to derive conditions on the optimal choice of the rotation period irrespective of the parameter of interest.

An important feature of (10) is that the weights in the linear combination, ξ_j $(j=0,\ldots,J)$, determine the numerator within the integral while the choice of r affects the denominator only. In Figure 1 we have plotted the inverse of the denominator, $f^{-1}(\lambda)$, for rotation periods r = 1, 2, 3, 4, 8, 12 assuming that $\rho = .5$. Similarly the numerator in (10), $g(\lambda)$, is presented in Figure 2 for six important special cases: estimation of the period means themselves $(J = 0;\ \xi_0 = 1)$, of differences in means between two successive periods $(J = 1;\ \xi_0 = 1,\ \xi_1 = -1)$ and of a k period sum or average $(J = k-1;\ \xi_j = 1,\ j = 0,\ldots,k-1)$ for k = 2, 3, 6 and 12. Note that both $f(\lambda)$ and $g(\lambda)$ are symmetric in λ and are therefore plotted for nonnegative values of λ only.

It is obvious from these figures and well known in the literature (see e.g. Cochran [1978, p. 348 ff.]) that the choice of the rotation period which minimizes the variance of the efficient estimator will in general depend on the linear combination of the means to be estimated. If the number of observations per wave does not depend on the choice of the rotation period ($n_r = n$ for all r), a series of cross sections (r = 1) will be optimal if a twelve period average is to be estimated because it is mainly the behaviour of $f^{-1}(\lambda)$ for small values of λ ("low frequency") which is important. If a difference in means is to be estimated a large value of r is optimal because the "high frequency" components dominate the variance.

Using (10) it is possible for any given ξ to compute the variance of the efficient estimator of $\xi'\mu$ given r and ρ. Computation of the integral in (10) is straightforward if the Residue Theorem is used (see e.g. Holland [1980, p.160]) after substituting $z = e^{i\lambda}$ and integrating over z on the unit circle. This yields that

Figure 1. The inverse of the denominator for several rotation periods with $\rho = .5$

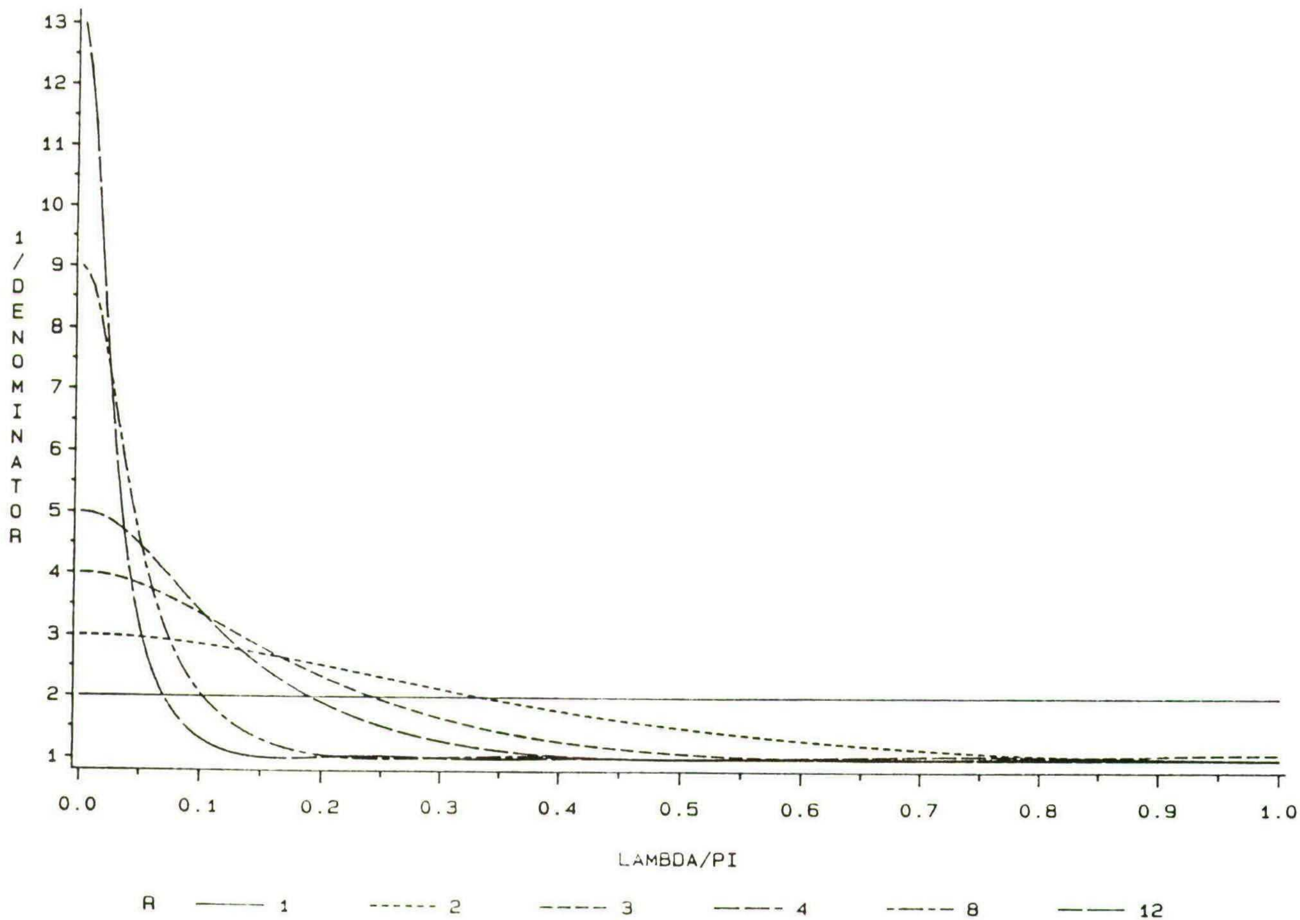
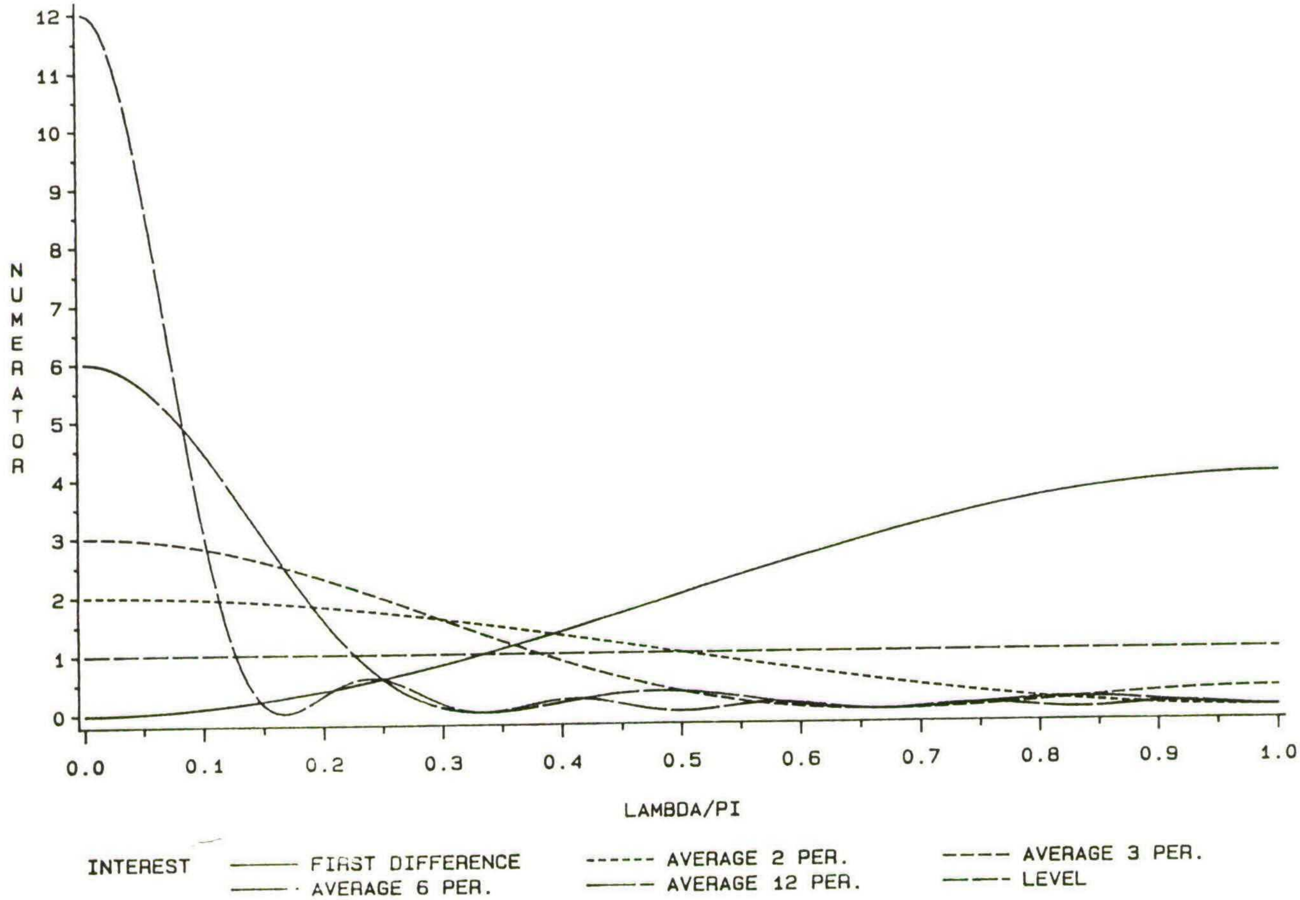


Figure 2. Values of numerator for several linear combinations of interest



$$V_r(\xi'\hat{\mu}) = \mathrm{Re}\,\left(\frac{1}{n_r}\,\sigma_\varepsilon^2 \sum_{j=1}^{r-1} \frac{2\, z_j^{r-2} \sum_{k=0}^{J} w_k z_j^k}{h'(z_j)}\right), \tag{11}$$

where $\mathrm{Re}(z)$ denotes the real part of z and the z_j are the $r-1$ zeroes within the unit circle of $h(z) = z^{r-1}f(z)$, and $h'(z) = \partial h(z)/\partial z$. The w_k and $f(z)$ are defined in (9) and (5) respectively. Note that the z_j are the $r-1$ roots of the lag polynomial of the moving average process introduced in Section 2 and that in (11) it is assumed for simplicity that $h(z) = 0$ has no multiple roots.

In order to compute the variances using (11) one has to determine the zeroes of a polynomial of degree $2r-2$. Although analytical results for $r = 3$ and $r = 4$ can be obtained they are not very revealing. Therefore we present analytical results for $r = 1$ and $r = 2$ only. For $r = 1$ the variance of $\xi'\hat{\mu}$ is seen to equal

$$V_1\{\xi'\hat{\mu}\} = \frac{1}{n_1}\sigma^2\, w_0 \tag{12}$$

where $\sigma^2 = \sigma_\alpha^2 + \sigma_\varepsilon^2$, and for $r = 2$ (11) reduces to

$$V_2\{\xi'\hat{\mu}\} = \frac{1}{n_2}\sigma^2 \surd(1-\rho^2)\,\left[\, w_0 + 2\sum_{j=1}^{J} w_j \left\{\frac{1-\surd(1-\rho^2)}{\rho}\right\}^j\,\right]. \tag{13}$$

Using (12) and (13) it is straightforward to check that lower bounds on n_1/n_2 for $r = 1$ to be preferable to $r = 2$ are given by

$$n_1/n_2 > \frac{1}{\surd(1-\rho^2)} \tag{14}$$

if one is interested in estimation of levels ($\xi_0 = 1;\ J = 0$); to

$$n_1/n_2 > \frac{1}{\surd(1-\rho^2)}\,\frac{\rho}{\rho - (1-\surd(1-\rho^2))} \tag{15}$$

if one is interested in estimation of first differences ($\xi_0 = 1$, $\xi_1 = -1$; $J = 1$); and to

$$n_1/n_2 > \frac{1}{\sqrt{(1-\rho^2)}} \frac{\rho}{\rho + (1 - \sqrt{(1-\rho^2)})} \tag{16}$$

when a sum or average over 2 periods is estimated ($\xi_0 = \xi_1 = 1$; $J = 1$). In Figure 3 we have drawn these bounds and indicated where $r = 1$ or $r = 2$ is preferable. The lines marked "all" will be dealt with in the next section. Bounds on n_1/n_2 for other linear combinations of interest can be obtained directly from (12) and (13). It can easily be verified e.g. that the condition in (14) is sufficient for optimality of a series of cross sections over $r = 2$ if one is interested in weighted averages of the period means with non-negative weights only as in that case $w_j > 0$ ($j = 0,..,J$).

Bounds similar to (14), (15) and (16) for other combinations of rotation periods can easily be obtained numerically using equation (11). In Figure 4 such bounds are presented for the case where the choice is restricted to $r = 1$ or $r = 3$. Evidently, $r = 1$ is not an attractive choice if the individual effect is dominant unless the number of observations in the cross sections is much larger than in the rotating panels.

Pairwise comparisons of rotation periods as presented above are relevant if one considers analysing either one or another existing data set in order to estimate the parameter of interest. If the sample still has to be drawn it is more natural not to restrict the choice of rotation periods to either one rotation period or another, but rather to a range of rotation periods with some prescribed maximum, r^{max}. Such a maximum will often have to be imposed, for example to avoid so called panel-effects or panel conditioning (behavioural changes because one participates too long), or because measurement errors or (non-random) non-response will increase if units are interviewed a larger number of periods. In order to model this choice of a rotation period, assume that the researcher or the data collecting agency is free to choose the rotation period r given some budget constraint. Let p_1 denote the cost of observing an individual for the first time and p_2 of observing it for a second time. Assume for simplicity that observing it for a third, fourth etc. time is equally expensive as the

Figure 3. Comparison of a series of cross sections (r = 1) and a rotating panel with r = 2

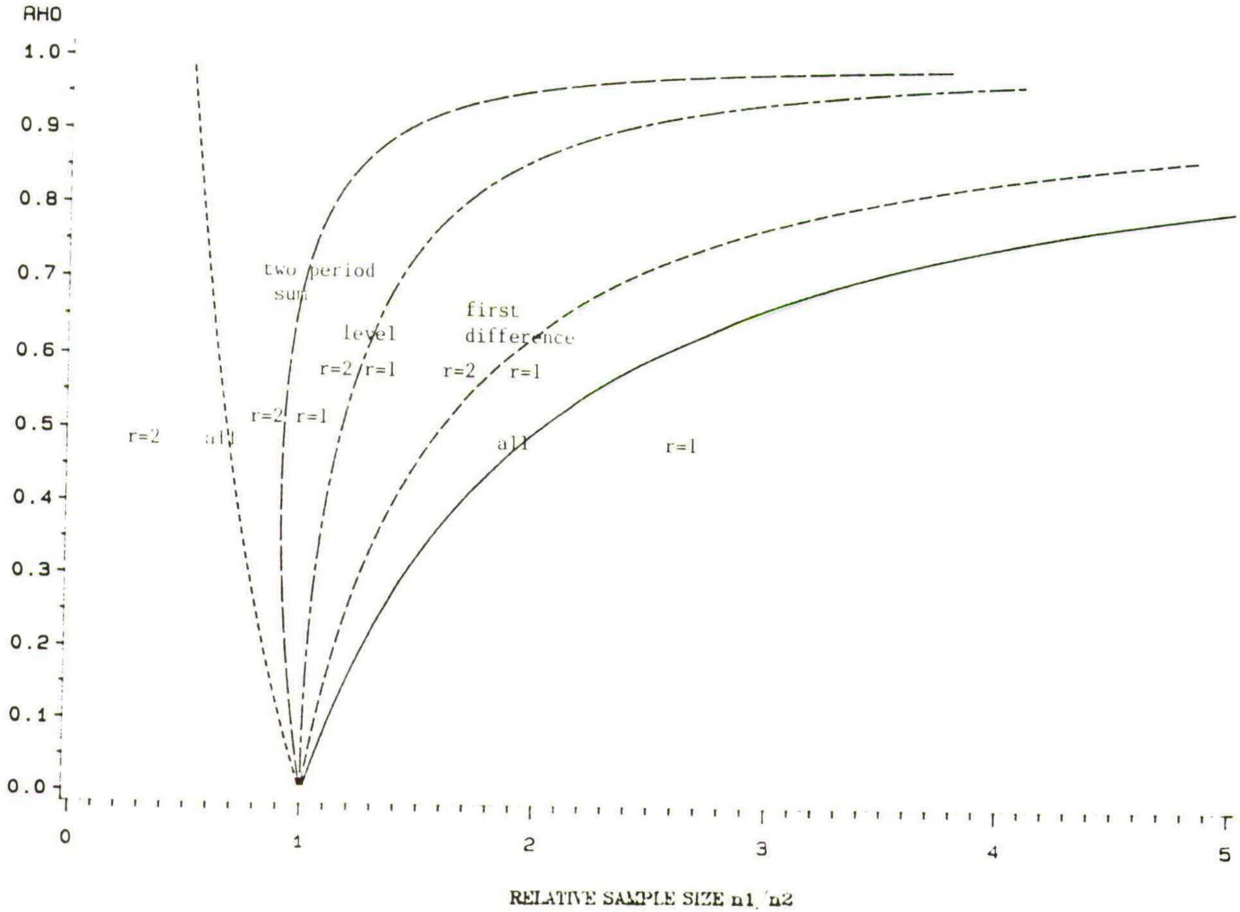
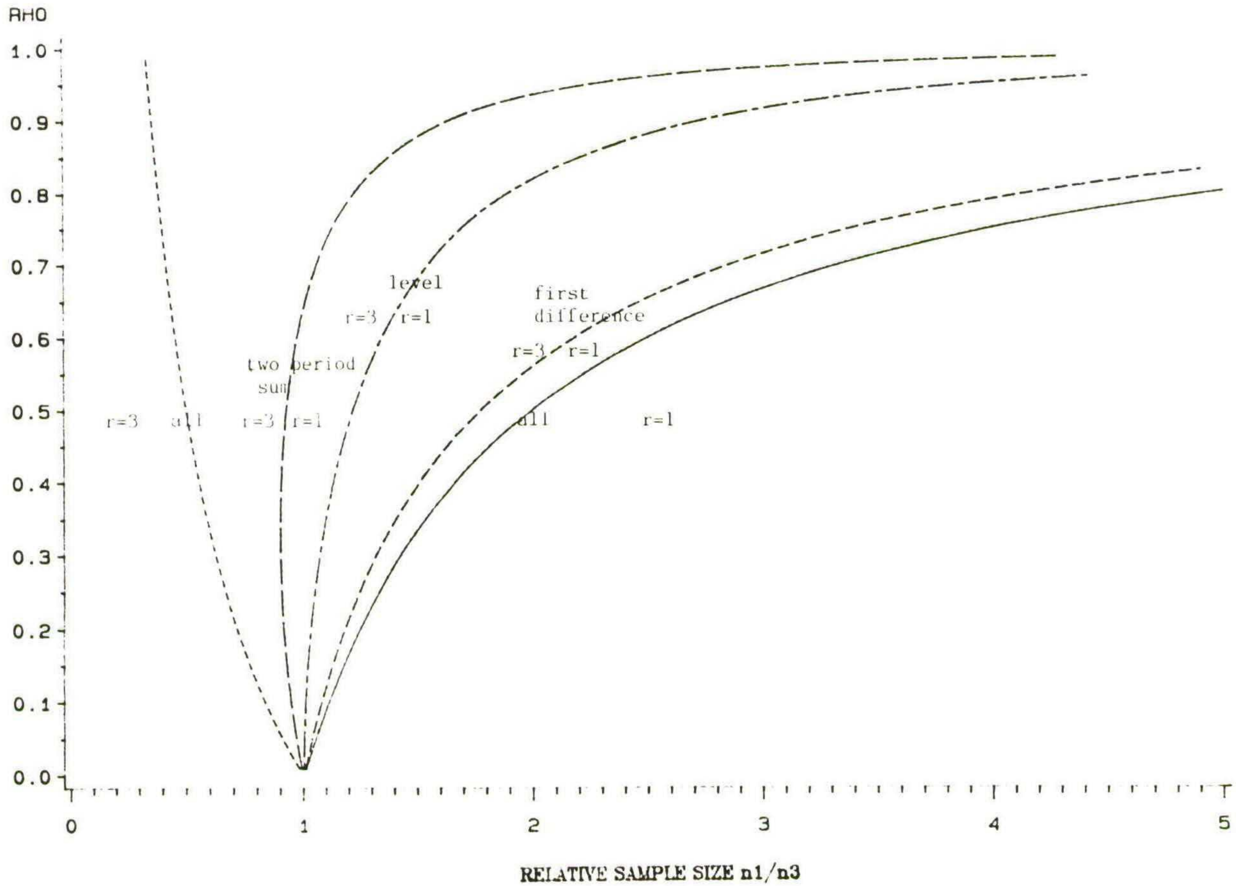


Figure 4. Comparison of a series of cross sections ($r = 1$) and a rotating panel with $r = 3$



second observation. If there is a budget B for each period the number of observations per wave in case of rotation period r equals

$$n_r = \frac{r\,B}{p_1 + (r-1)p_2} = \frac{r\,B^*}{1 + (r-1)\alpha} \tag{17}$$

where $B^* = B/p_1$ and $\alpha = p_2/p_1$, the relative cost of a repeated observation. Typically, α is smaller than unity.

If $r^{max} = 2$ the choice is again restricted to either $r = 1$ or $r = 2$ and the bound in (16) for example, can easily be rewritten to show that spending the budget on a series of cross sections will yield more precise estimates of averages over two periods than a rotating panel with rotation period equal to two if

$$\alpha > \frac{2}{\surd(1-\rho^2)} \; \frac{\rho}{\rho + (1 - \surd(1-\rho^2))} - 1. \tag{18}$$

Note that (14) and (15) imply that the rotating panel will always be preferable if levels or period to period changes are to be estimated as long as $\alpha < 1$ which is likely to be the case.

The choice of the rotation period if $r^{max} = 8$ is visualized in Figures 5, 6, 7 and 8 for the case of averages over 2, 3, 6 and 12 periods respectively. Note that no figures are included for the estimation of a single period mean or a difference in means as our numerical results suggest that in these cases the largest rotation period will always be optimal.

However one should not be tempted to think that a true panel ($r = \infty$) would yield even more efficient estimates if the preferred choice for the rotation period is r^{max}. In case of equal sample sizes, for example, a true panel will yield estimators of the period means which are as efficient as the ones derived from a series of cross sections (see e.g. Cochran [1977, p.345 ff.]). In general, Figures 5 to 8 clearly show that intermediary rotation periods ($r = 2,\ldots,7$) are optimal in very small parts of the (ρ,α) space only. Usually it will either be the maximal ($r = 8$) or the minimal ($r = 1$) rotation period which is optimal.

Figure 5. The optimal rotation period for estimating a two period average mean

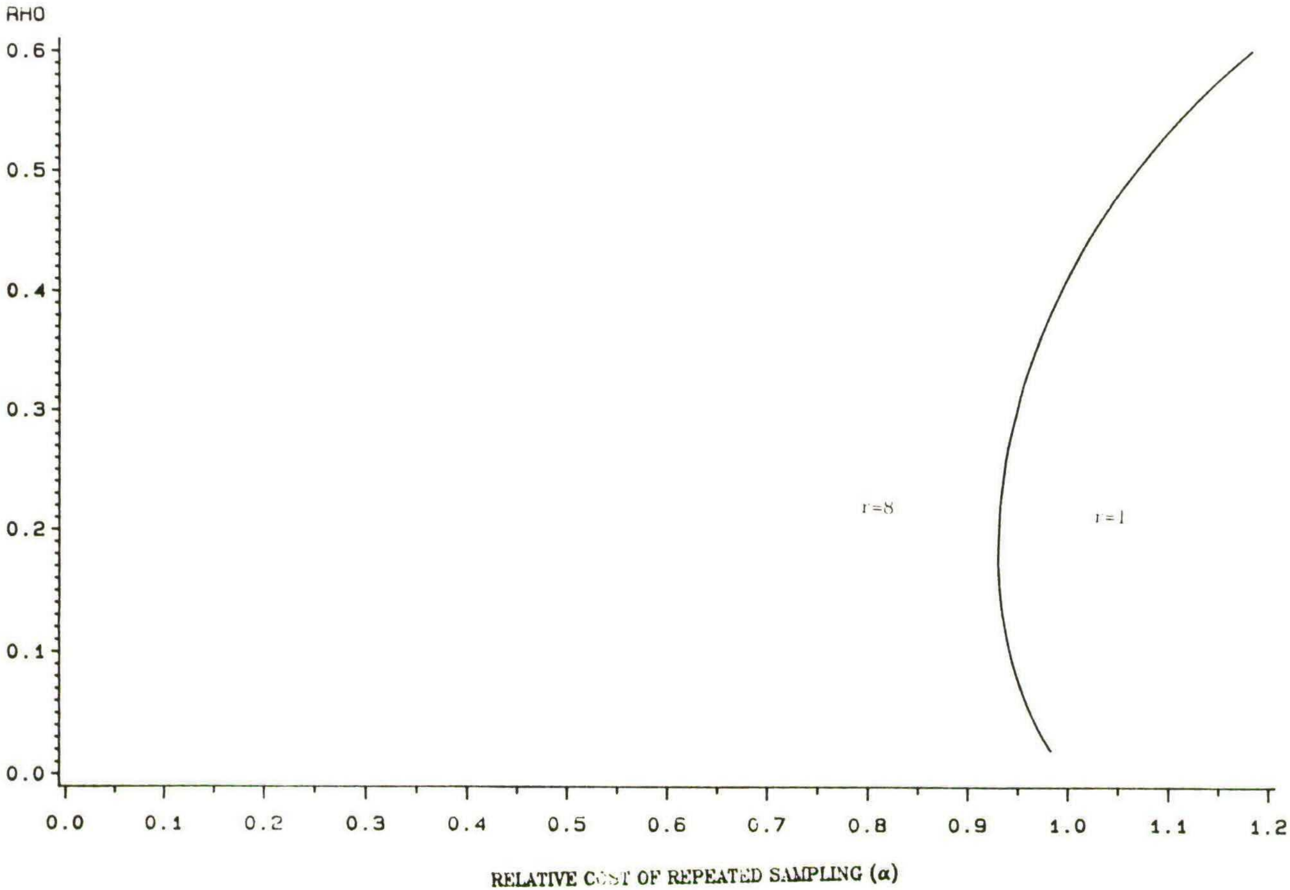


Figure 6. The optimal rotation period for estimating a three period average mean

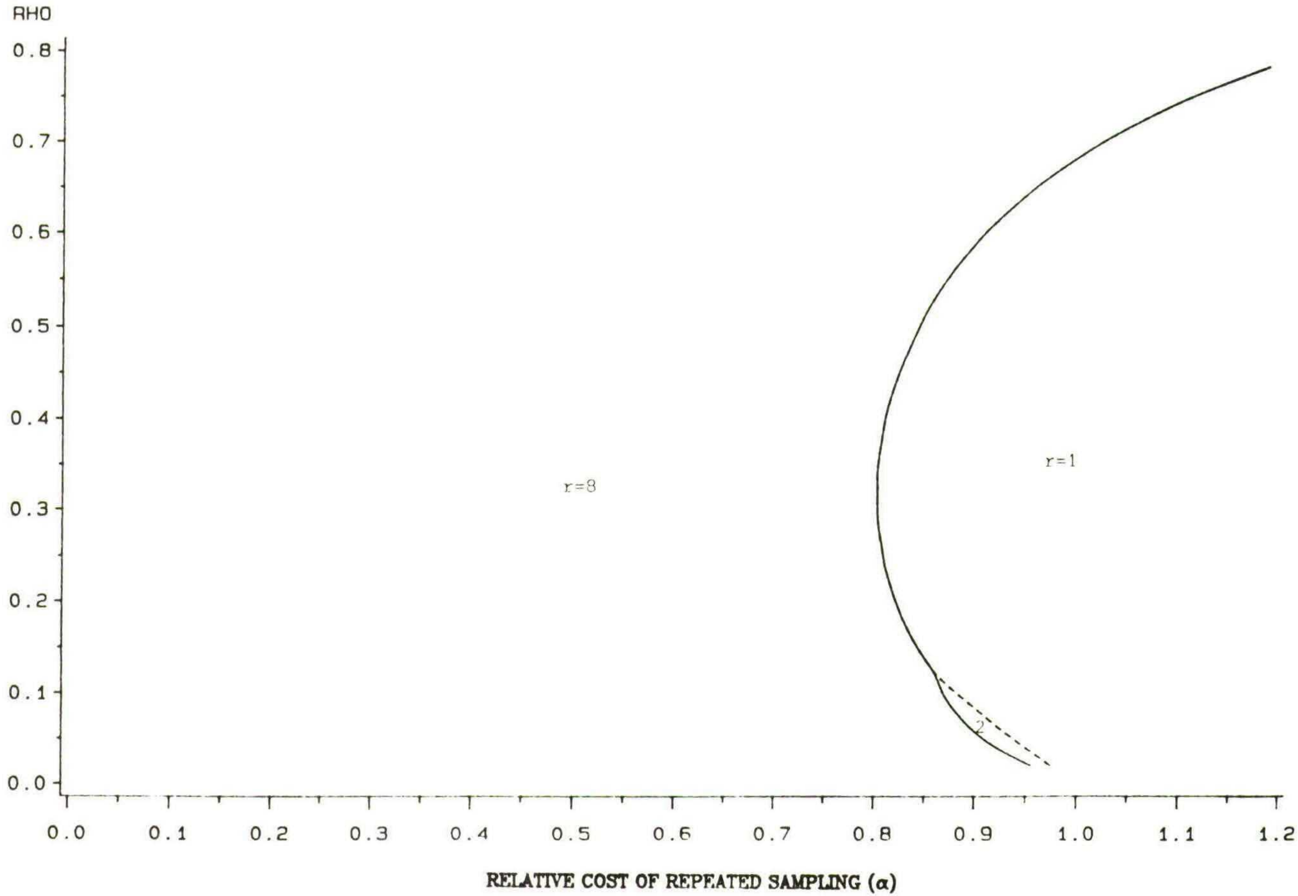


Figure 7. The optimal rotation period for estimating a six period average mean

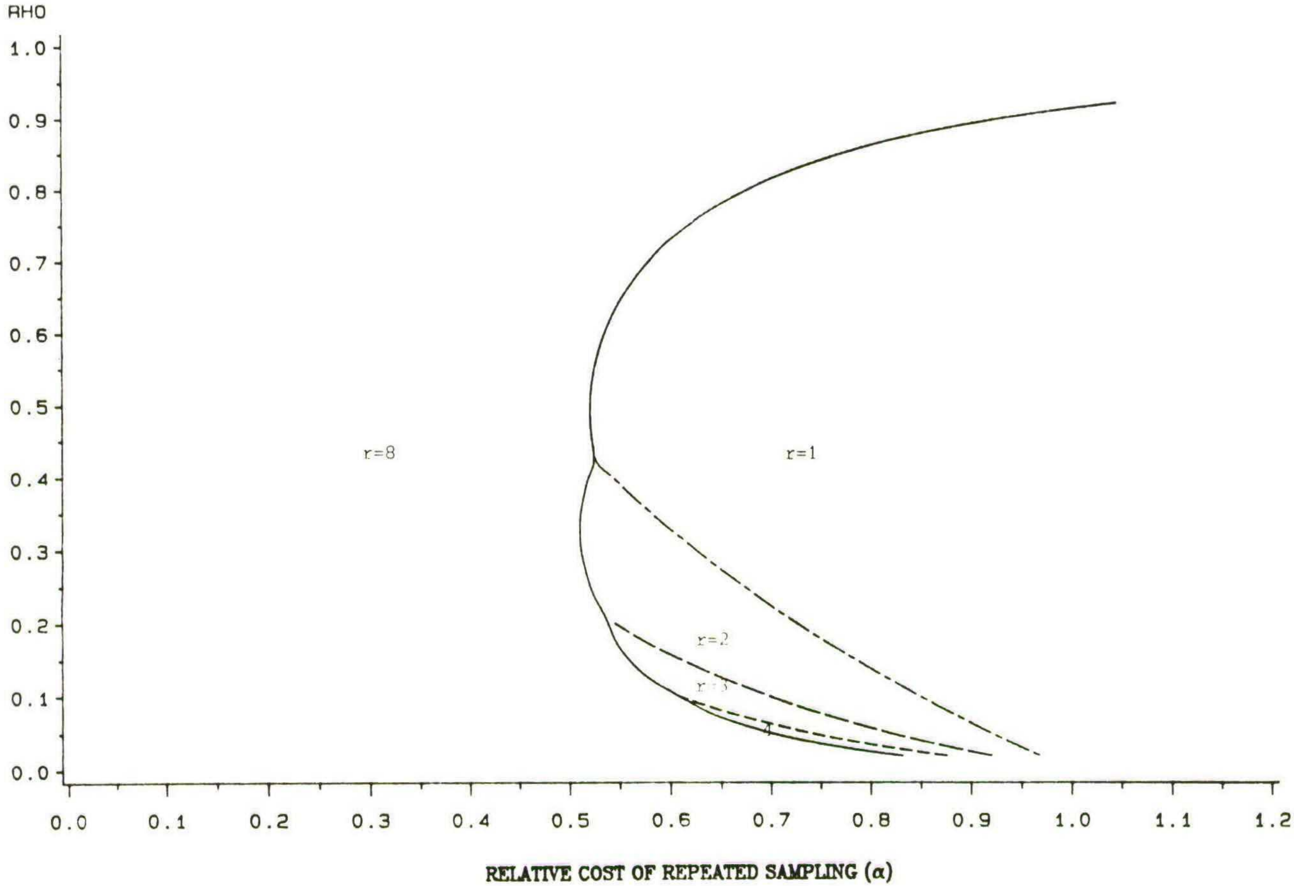
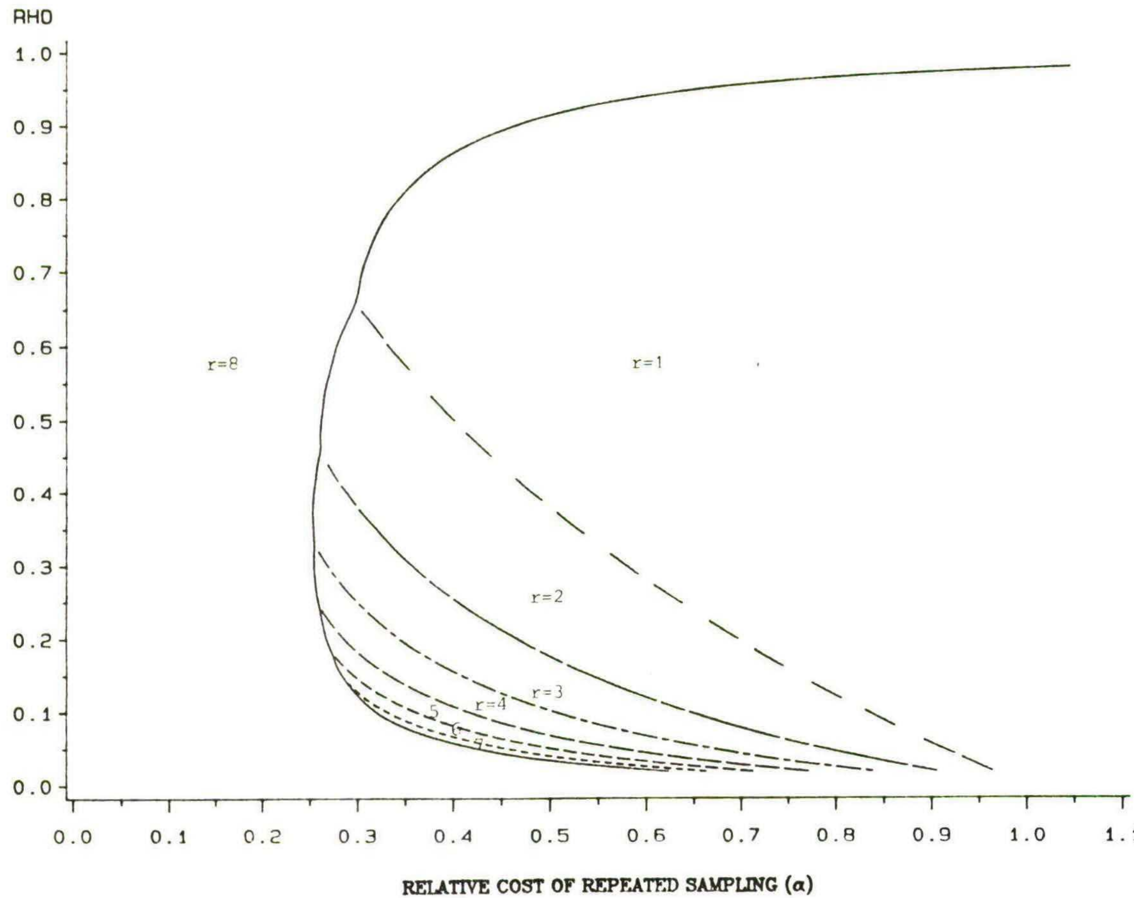


Figure 8. The optimal rotation period for estimating a twelve period average mean



It is not only relevant to know how the optimal rotation period can be determined, but also to know how much efficiency will be lost if a sub-optimal choice is made. In Table 1 we present the relative efficiencies compared with a series of cross sections $(r = 1)$ for several rotation periods, some specific parameters of interest and values of α and ρ. As an illustration consider the case where $\rho = .6$ and $\alpha = 1$ (equal cost). Then it follows from Table 1 that the variance of the estimator of a particular μ_t in case $r = 4$ is equal to 71 % of the variance for $r = 1$ (a series of cross sections), and only 43 % if one is estimating a first difference. So it will be clear that the gains in efficiency can be quite substantial if an optimal sample design is chosen. Even in case of equal cost $(\alpha = 1)$ gains of more than 50 % are not uncommon.

4. The optimal choice of the rotation period irrespective of the parameter of interest

A problem with the fact that the optimal choice of the rotation period generally depends on the aim for which the rotating panel is to be used is that one usually wants to use one panel for the estimation of both levels, differences and averages. In this section we will therefore analyse under what conditions the optimal value of the rotation period can be determined irrespective of the linear combination of the means to be estimated. Evidently, from (10), a panel with rotation period r will yield a more efficient estimator of $\xi'\mu$ than another panel with rotation period s irrespective of the choice of ξ if

$$n_r^{-1} f_r^{-1}(\lambda) < n_s^{-1} f_s^{-1}(\lambda), \qquad -\pi < \lambda \leq \pi \tag{19}$$

where subscripts are added to the function $f(\lambda)$ to stress its dependence on the rotation period. If the value η_{rs} is defined by

$$\eta_{rs} = \max_{\lambda \in [-\pi, \pi]} f_s(\lambda) f_r^{-1}(\lambda), \tag{20}$$

Table 1. Relative efficiency for a panel with rotation period r compared with a series of cross sections

		levels		first difference		2 period sum	
		$\alpha=.5$	$\alpha=1$	$\alpha=.5$	$\alpha=1$	$\alpha=.5$	$\alpha=1$
	$\rho=.3$	.71	.95	.61	.81	.83	1.10
$r=2$	.6	.60	.80	.40	.53	.80	1.07
	.9	.33	.44	.12	.16	.53	.71
	$\rho=.3$	.62	.93	.50	.76	.74	1.11
$r=3$	.6	.49	.74	.31	.46	.68	1.02
	.9	.24	.37	.08	.12	.41	.61
	$\rho=.3$	.57	.92	.46	.73	.68	1.10
$r=4$	.6	.44	.71	.27	.43	.61	.98
	.9	.21	.33	.07	.11	.34	.55
	$\rho=.3$	.49	.88	.40	.71	.59	1.05
$r=8$	.6	.36	.64	.23	.41	.49	.86
	.9	.15	.27	.06	.10	.24	.43
	$\rho=.3$	.46	.86	.38	.70	.55	1.01
$r=12$	.6	.33	.60	.22	.40	.43	.80
	.9	.13	.24	.05	.10	.20	.38

equation (19) implies that the panel with rotation period r will yield more efficient estimators of any linear combination of the period means in (1) than a panel with rotation period s if the numbers of observations per wave satisfy $n_r/n_s > \eta_{rs}$.

Let us first of all consider the choice between a series of cross sections and a rotating panel with rotation period equal to two. Using

$$f_1(\lambda) = \frac{1}{2\pi}(1 - \rho) \tag{21}$$

and

$$f_2(\lambda) = \frac{1}{2\pi}(1 + \rho)^{-1}(1 - \rho\cos\lambda) \tag{22}$$

it is straightforward to verify that $\eta_{12} = (1 - \rho)^{-1}$ and $\eta_{21} = 1 + \rho$. If $n_1 > (1 - \rho)^{-1}n_2$ a series of cross sections is preferable to a rotating panel with rotation period equal to two without ambiguity, while the opposite is true if $n_1 < (1 + \rho)^{-1}n_2$. If neither of these conditions is satisfied the choice of the optimal design should depend on the parameters of interest.

Using (6) it can be shown that $\eta_{1s} = (1 - \rho)^{-1}$ $(s = 2, 3, ..)$ and that $\eta_{rs} = \{1 + (r-1)\rho\} / \{1 + (s-1)\rho\}$ if $r > s$. In more general cases it does not appear to be possible to obtain simple analytical expressions for η_{rs}, but it is of course straightforward to maximize (20) numerically. Numerical results for three different values of ρ are presented in Table 2. If, for example, $\rho = .3$, $r = 3$ will be unambiguously preferable to $r = 2$ if $n_3/n_2 >$ 1.23, while $r = 2$ will be unambiguously preferable to $r = 3$ if $n_2/n_3 > 1.22$, i.e. if $n_3/n_2 < .82$. It is evident from these results that it is relatively simple to choose the optimal rotation period if ρ is small, which implies that individual effects are not very important in the analysis of variance model (1). Of course the choice of the rotation period is also less important if ρ is small since the obtainable efficiency gain will be small in that case.

In Figure 3 where we restrict ourselves to the choice between $r = 1$ and $r = 2$, we have drawn the bounds η_{12} and η_{21}^{-1} on the relative sample size n_1/n_2. These bounds (marked "all") determine regions in which $r = 1$ and

Table 2. Lower bounds η_{rs} on quotient of number of observations per wave n_r/n_s for panel with rotation period r to be unambiguously preferable

		s=1	s=2	s=3	s=4	s=8	s=12
	ρ=.3	-	1.43	1.43	1.43	1.43	1.43
r=1	.6	-	2.50	2.50	2.50	2.50	2.50
	.9	-	10.00	10.00	10.00	10.00	10.00
	ρ=.3	1.30	-	1.22	1.38	1.67	1.76
r=2	.6	1.60	-	1.50	1.92	2.95	3.40
	.9	1.90	-	1.99	3.06	7.16	10.29
	ρ=.3	1.60	1.23	-	1.18	1.67	1.92
r=3	.6	2.20	1.38	-	1.35	2.61	3.48
	.9	2.80	1.47	-	1.60	4.54	7.67
	ρ=.3	1.90	1.46	1.19	-	1.55	1.92
r=4	.6	2.80	1.75	1.27	-	2.14	3.15
	.9	3.70	1.95	1.32	-	3.04	5.55
	ρ=.3	3.10	2.38	1.94	1.63	-	1.42
r=8	.6	5.20	3.25	2.36	1.86	-	1.69
	.9	7.30	3.84	2.80	1.97	-	1.99
	ρ=.3	4.30	3.31	2.69	2.26	1.39	-
r=12	.6	7.60	4.75	3.45	2.71	1.46	-
	.9	10.90	5.74	3.89	2.95	1.49	-

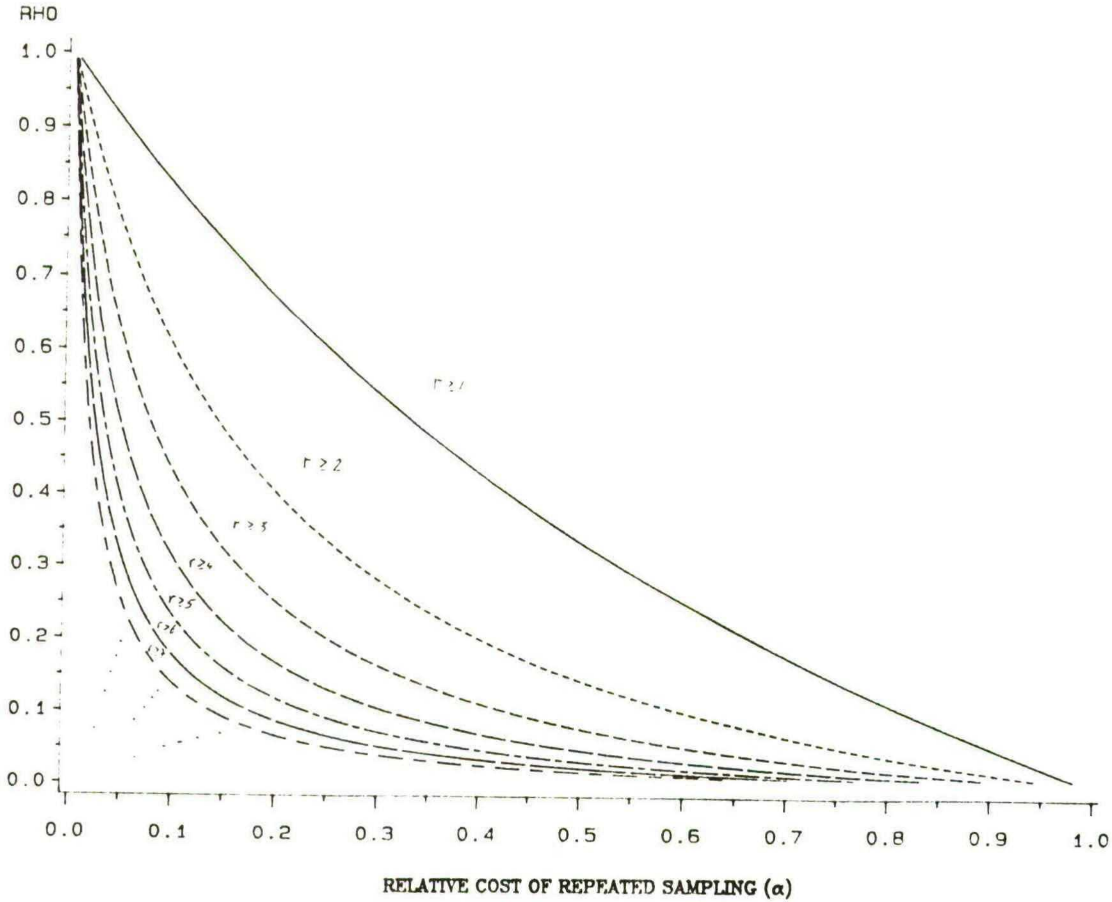
$r = 2$ respectively are unambiguously preferable to the other. Analogously, bounds for the choice between $r = 1$ and $r = 3$ are drawn in Figure 4. Of course the bounds derived in Section 3 for some specific linear combinations of interest will always lie in the region where the choice depends on the parameter of interest.

Similar to the approach chosen in Section 3, we have used the results on pairwise comparisons of rotating samples irrespective of the parameter of interest to obtain results on the choice of a rotation period r restricted by some r^{max} assuming that the cost structure (17) holds. Using (17) each bound η_{rs} can be rewritten as a bound on the relative cost of resampling, α. Pairwise comparisons are used to determine regions of the parameter space where one or more rotation period(s) can never be optimal, whatever the parameter of interest. These regions are presented in Figure 9 where we assumed $r^{max} = 8$. Note that in most regions there is no unique optimal rotation period since this will depend on the parameter of interest. However, optimality of some rotation periods can be excluded for some values of ρ and α. If e.g. $\rho < (1-\alpha)/(1+\alpha)$ a series of cross sections will not be optimal for any choice of the parameter of interest. More general results can be inferred from Figure 9. If e.g. $\alpha < .5$ a series of cross sections cannot be optimal for any parameter of interest if $\rho < .33$ while $r = 2$ and $r = 3$ will always be suboptimal if $\rho < .17$ and $\rho < .09$ respectively.

5. The optimal design for specific parameters of interest if one is estimating in recent periods

A drawback of the results of the previous sections is that they are only valid if one is estimating period means not too close to the end of the sample period. In those sections we restricted attention to the limiting case where the number of future periods on which data are available, S, tends to infinity. In this section we consider the case of a fixed S, still assuming for convenience that the number of past periods in the sample, T,

Figure 9. Regions with restrictions on the optimal rotation period r



is infinitely large. The results in this section suggest that unless ρ is close to unity and S very small the earlier results are hardly affected.

The main reason for considering the limiting case where S tends to infinity in the previous sections is that in this case a simple expression for the inverse of the matrix A, which arises in the variance covariance matrix of the efficient estimator, is available. In this section we show how to obtain an expression for the inverse of this matrix if S is fixed. Denote the moving average process which generates the autocovariances $E\, x_t x_{t+\tau} = a_{|\tau|}$ by $x_t = \vartheta(L)\, e_t$ with $e_t \sim NID(0, \sigma_e^2)$ as before. Define $z_t = \vartheta^{-1}(L)\, e_t = \psi(L)\, e_t$ where $\psi(L) = \psi_0 + \psi_1 L + \psi_2 L^2 + \ldots$. In Section 2 where T and S tended to infinity we have approximated the inverse of the matrix Σ^{MA} defined by $\Sigma^{MA}_{ts} = E\, x_t x_s$ by Σ^{AR} defined as $\Sigma^{AR}_{ts} = E\, z_t z_s$. A valid approximation to $(\Sigma^{MA})^{-1}$ if S is fixed is to use the matrix of covariances of more than S period ahead prediction errors of the AR process instead of the matrix of covariances of the variable z_t itself. In the appendix we show that if we define the symmetric matrix $B = (b_{\ell k})$ by

$$b_{\ell k} = \sigma_e^{-2} \sum_{j=0}^{S-k} \psi_j \psi_{j+k-\ell} \qquad (k \geq \ell;\ -T \leq \ell,\ k \leq S) \tag{23}$$

and partition B as

$$B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \tag{24}$$

where B_{22} has dimension $(r-1)\times(r-1)$, it holds true that, if $\xi_j = 0$ for $j > J$ for some fixed J,

$$\lim_{T\to\infty} \xi' A^{-1} \xi = \lim_{T\to\infty} \xi' \{ B - \begin{bmatrix} B_{12} \\ B_{22} \end{bmatrix} (I + P_{22} B_{22})^{-1} P_{22} \begin{bmatrix} B_{21} & B_{22} \end{bmatrix} \} \xi \tag{25}$$

where S is fixed and P_{22} denotes the lower right $(r-1)x(r-1)$ block of $A - \Sigma^{MA}$. Evidently, (25) generalizes (4).

In order to illustrate these results consider again the case where $r = 2$ in which case $\vartheta(L) = 1 - \vartheta L$ with

$$\vartheta = \frac{1 - (1 - \rho^2)^{1/2}}{\rho} \text{ and } \sigma_e^2 = \frac{1}{2}\frac{\rho}{\vartheta}\frac{1}{1+\rho} . \tag{26}$$

In this case $b_{\ell k}$ reduces to

$$b_{\ell k} = \sigma_e^{-2} \sum_{j=0}^{S-k} \vartheta^j \vartheta^{j+k-\ell} = \sigma_e^{-2} \vartheta^{k-\ell} \frac{1 - \vartheta^{2(S-k+1)}}{1 - \vartheta^2} \quad (k \geq \ell). \tag{27}$$

Using (23) and (25) it is straightforward to verify that the variance of $\xi'\hat{\mu} = \Sigma_{j=0}^{J} \xi_j \hat{\mu}_{-j}$ for a rotating panel with rotation period $r = 2$ is given by

$$V_2\{\xi'\hat{\mu}\} =$$

$$\frac{1}{n_2} \sigma^2(1-\rho) \left\{ \sum_{\ell=0}^{J} \sum_{k=0}^{J} \xi_\ell \xi_k b_{-k,-\ell} + \frac{1}{2} \frac{\rho^2}{(1-\rho\vartheta)(1+\rho)} \left[\sum_{\ell=0}^{J} \xi_\ell b_{S,-\ell} \right]^2 \right\}. \tag{28}$$

Note that (28) is a generalization of (13), while the two expressions coincide if S tends to infinity. For the special case of estimating the period mean μ_0, it is readily verified from (28) that a lower bound on n_1/n_2 for $r = 1$ to be preferable to $r = 2$ is given by

$$n_1/n_2 > (1-\rho^2)^{-1/2} \left\{ 1 + 2 \left[\vartheta^{2S+2} - \frac{\vartheta}{\rho} \sqrt{(1-\rho^2)} \sum_{s=S+1}^{\infty} \vartheta^{2s} \right] \right\}^{-1}. \tag{29}$$

Note that this bound is always higher than the one given by (17) in the case of estimating an individual mean when the number of future observations tends to infinity. This is not surprising as the non-availability of future observations has no impact on the efficient estimator in the cross section case ($r = 1$), but implies an information loss for the $r = 2$ case. Moreover, if S tends to infinity (29) tends to (14) since $|\vartheta| < 1$ for $\rho < 1$.

For the special case of $S = 0$ (29) reduces to

$$n_1/n_2 > \frac{1}{\sqrt{(1-\rho^2)}} \left[\frac{1}{2} \frac{\rho^2}{1-\sqrt{(1-\rho^2)}} \right]. \tag{30}$$

This bound is always lower than the one given by Eckler [1955] (in his "two level rotation sampling" case), viz.

$$n_1/n_2 > \frac{1}{\surd(1-\rho^2)} \left[\frac{1}{2} \right]. \tag{31}$$

This result is not very surprising since Eckler assumes that *all* individuals in the sample are observed twice, which implies that the final period sample size is half of the sample size in the preceeding periods which is not very natural.

In Table 3 we present values for the lower bounds on the relative sample size n_r/n_1 for rotation period r to be preferable to a series of cross sections for r = 2, 3, 4 and three specific parameters of interest. If the sample sizes n_r and n_1 coincide ($n_r = n_1$), the values in the table can also be interpreted as a measure of the relative efficiency. The value of S indicates how many periods of observation are available after the estimation. S = 0 characterizes the case one is estimating in the final period, while S = ∞ yields the results of Section 3.

Table 3 shows that the differences in the bounds are rather small, especially for small and moderate ρ. Moreover, the more observations are available after the estimation period(s), the smaller the difference between the exact bounds and the bounds from Section 3 will be. Table 3 therefore clearly suggests that when ρ is known to be moderate the results of Section 3 may be used as an approximation.

It is clear from Table 3 that, if the cost structure in (17) is valid and the relative cost of resampling α is smaller than unity, a rotating panel will be preferred to a series of cross sections, when one is interested in a level as well as a first difference. If α is still smaller (e.g. .8) then the rotating panel is also preferable in case of estimation of a two period sum.

Table 3. Relative efficiency for a panel with rotation period r compared with a series of cross sections (in case of equal sample sizes)

		$S = 0$	$S = 1$	$S = 2$	$S = \infty$
level					
$r = 2$	$\rho = .3$	.98	.96	.95	.95
	.6	.89	.85	.82	.80
	.9	.61	.58	.54	.44
$r = 3$	$\rho = .3$	.96	.95	.93	.93
	.6	.85	.79	.75	.74
	.9	.53	.47	.43	.37
$r = 4$	$\rho = .3$	.95	.94	.93	.92
	.6	.82	.78	.74	.71
	.9	.48	.45	.42	.33
first difference					
$r = 2$	$\rho = .3$	.82	.81	.81	.81
	.6	.55	.54	.53	.53
	.9	.17	.17	.16	.16
$r = 3$	$\rho = .3$	.77	.76	.76	.76
	.6	.48	.46	.46	.46
	.9	.13	.13	.12	.12
$r = 4$	$\rho = .3$	.75	.74	.74	.73
	.6	.45	.44	.44	.43
	.9	.12	.11	.11	.11
two period sum					
$r = 2$	$\rho = .3$	1.12	1.10	1.10	1.10
	.6	1.15	1.08	1.07	1.07
	.9	.94	.80	.74	.71
$r = 3$	$\rho = .3$	1.14	1.12	1.11	1.11
	.6	1.16	1.08	1.04	1.02
	.9	.87	.78	.71	.61
$r = 4$	$\rho = .3$	1.15	1.13	1.11	1.10
	.6	1.15	1.08	1.03	.98
	.9	.81	.75	.70	.55

6. Concluding remarks

The collection of data, e.g. in consumer surveys, is characterized by its high cost. Therefore it is profitable to obtain the highest level of information with the lowest attainable cost by choosing an optimal sample design. In this paper we have analysed the question which rotation period in a rotating panel is optimal in the sense that it yields the most efficient estimators of specific linear combinations of the period means or any linear combination of the period means in a simple analysis of variance model.

The analysis of variance model (1) is characterized by an individual effect α_i, implying a constant correlation over time between different observations on the same unit. The results can however easily be extended to more general correlation patterns such as the exponentially declining patterns considered by Eckler [1955] and Patterson [1950], because the assumptions on the correlation pattern do not affect the structure of the band matrix A to be inverted in order to derive expressions for the variance of efficient parameter estimates.

In a previous paper (Nijman and Verbeek [1988]) where we discussed the choice between a pure panel, a pure cross section and a combination of these two data sources, model (1) was used to model monthly consumer expenditures on food and clothing in 1985 using the so called Expenditure Index panel conducted by INTOMART, a private marketing research agency. The assumptions on the error terms appeared to be valid and the maximum likelihood estimates of ρ in (1) for food and clothing were .76 and .25 with standard errors .005 and .002 respectively.

These results imply that a series of cross sections cannot be optimal to monitor expenditures on clothing if the relative cost of resampling is less than .60 irrespective of the parameter of interest. The corresponding figure for food where the individual effect is more prominent is .14. If one considers one parameter of interest only these bounds can be sharpened. Suppose e.g. that one is interested in estimates of the average consumer expenditures in the last month of observation. Our results in Section 5 imply that a rotating panel with rotation period 2 is preferable to a series of cross sections if $n_2/n_1 > .79$ for food and $n_2/n_1 > .98$ for clothing. If we want to estimate average expenditures in a more distinct past these bounds relax to $n_2/n_1 > .65$ and $n_2/n_1 > .97$ respectively. Alternatively, if one is interested in a change in means then a rotating panel with $r = 2$ is already preferable to a series of cross sections if $n_2/n_1 > .37$ and $n_2/n_1 > .85$ for food and clothing respectively, while these bounds relax to $n_2/n_1 > .35$ and $n_2/n_1 > .85$ respectively if one is estimating in a more distinct past.

In summary, our results show that the gains from choosing an optimal rotation design can be quite substantial, even in the case the cost of a repeated observation equals the cost of a first observation ($\alpha = 1$). Our analysis suggests that in many cases either the smallest ($r = 1$) or the highest possible rotation period is optimal. If the rotation period is not chosen optimally, the efficiency loss can be considerable, as will be clear from Table 3. In the above-mentioned example of food expenditures, a rotating panel with $r = 4$ will yield an efficiency gain of over 70 % if one is estimating a difference in subsequent means, compared to a series of independent cross sections with the same number of observations in every period.

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Appendix : Details on the technicalities

In this appendix we will derive the expression for A given by (3) and prove the results in (4), (6) and (25).

Proof of (3)

To obtain (3) we split the individuals in the data set into r independent subsamples, each of which containing a time series of independent small panels. If $r = 2$ e.g. a first subsample consists of the units included in the first wave only, of those included in the second and the third wave, of those in the fourth and the fifth wave, etc., while the second subsample consists of units observed in even periods and in the preceeding period. Then we use ordinary least squares to generate the efficient estimators $\hat{\mu}_j$ of μ in the j-th subsample ($j=1,\dots,r$). If we define a $k \times k$ matrix Ω_k by

$$\Omega_k = \sigma_\varepsilon^2 I_k + \sigma_\alpha^2 \iota_k \iota_k' \tag{A.1}$$

where I_k is the k dimensional identity matrix and ι_k is a k dimensional vector of ones, it can be easily verified that

$$\hat{\mu}_j \sim N(\mu, \frac{r}{n_r} V_j), \tag{A.2}$$

where V_j is a block diagonal matrix with upper left block Ω_j, subsequently $[(T-j)/r]$ blocks equal to Ω_r where $[x]$ denotes the integer part of x and finally a lower right block $\Omega_{T-j-[(T-j)/r]}$, and n_r/r is the number of observations per period in each subsample.

Since the $\hat{\mu}_j$ are independent the efficient estimator of μ using all subsamples is given by

$$\hat{\mu} = \left[\sum_{j=1}^{r} V_j^{-1} \right]^{-1} \sum_{j=1}^{r} V_j^{-1} \hat{\mu}_j \tag{A.3}$$

and this estimator satisfies

$$\hat{\mu} \sim N(\mu, \frac{\sigma_\varepsilon^2}{n_r} V) \quad \text{with } \sigma_\varepsilon^2 V = \left[\frac{1}{r} \sum_{j=1}^{r} V_j^{-1} \right]^{-1}. \tag{A.4}$$

Using the fact that

$$\Omega_k^{-1} = \sigma_\varepsilon^{-2} \left[I_k - \sigma_\alpha^2(\sigma_\varepsilon^2 + k\sigma_\alpha^2)^{-1} \iota_k \iota_k' \right] \tag{A.5}$$

it is easy to check that the elements of $A = V^{-1}$ satisfy equations (3).

Proof of (4)

Subsequently we want to prove equation (4) which states that

$$\lim_{T\to\infty} \Delta = \lim_{T\to\infty}(\xi' A^{-1}\xi - \xi'\Sigma^{AR}\xi) = 0 \tag{A.6}$$

if $\xi_j = 0$ for $|j| > J$ for some finite J.
First define Σ^{MA} and Σ^{AR} as in Section 2. Apart from the $(r-1)\times(r-1)$ upper left and lower right corners, A equals Σ^{MA}. Moreover, as stated in the main text, Shaman [1975] shows that Σ^{MA}, apart form the $(r-1)\times(r-1)$ upper left and lower right corners, is equal to $(\Sigma^{AR})^{-1}$. Define the symmetric $(2T+1)\times(2T+1)$ matrix S as $S = A - (\Sigma^{AR})^{-1}$. From the results above it is obvious that only the $(r-1)\times(r-1)$ upper left and lower right corners of S contain non zero elements. Since $(\Sigma^{AR})^{-1}$ is positive definite and S is symmetric there exists a nonsingular matrix Q such that

$$Q'(\Sigma^{AR})^{-1}Q = I \tag{A.7}$$

$$Q'SQ = D = \mathrm{Diag}\{\lambda_j\} \tag{A.8}$$

with D a diagonal matrix containing the eigenvalues λ_j of $\Sigma^{AR}S$ and Q the eigenvectors of $\Sigma^{AR}S$ (see e.g. Gantmacher [1959, p.310 ff.]). Using (A.7) and (A.8) it is easily verified that

$$\Delta = \xi' A^{-1}\xi - \xi'\Sigma^{AR}\xi = -\sum_{j=-T}^{T} \delta_j^2 \frac{\lambda_j}{1+\lambda_j} \quad \text{with } \delta = Q'\xi. \tag{A.9}$$

If the eigenvectors of $\Sigma^{AR}S$ associated with the zero eigenvalues are included in a matrix Q_1, and the remaining 2r-2 eigenvectors in a matrix Q_2, it is evident that $Q_2'(\Sigma^{AR})^{-1}Q_1 = 0$ and that the first and last r-1 rows of Q_1 consist of only zero elements. From (A.9) then follows the condition that

$$\lim_{T\to\infty} \Delta = 0, \quad \text{if} \quad \lim_{T\to\infty} Q_2'\xi = 0 ,$$

that is the first and last r elements of $\Sigma^{AR}\xi$ approach zero. Since $\xi_j = 0$, $|j| > J$ and Σ^{AR} is a covariance matrix of an autoregressive process, this last condition is satisfied.

Proof of (6)

We start with $\lambda \neq 0$. First note that, by using (3)

$$2\pi\, f(\lambda) = \sum_{\tau=-r+1}^{r-1} a_{|\tau|} e^{-i\lambda\tau} =$$

$$= \frac{1}{1+(r-1)\rho} \left[1 + (r-2)\rho - \frac{2\rho}{r} \sum_{k=1}^{r-1} k \left\{ e^{i(r-k)\lambda} + e^{-i(r-k)\lambda} \right\} \right] . \qquad \text{(A.10)}$$

Furthermore,

$$\sum_{k=1}^{r-1} k\, e^{i(r-k)\lambda} = e^{ir\lambda} \sum_{k=1}^{r-1} k \left(e^{-i\lambda}\right)^k =$$

$$e^{ir\lambda} \left[\frac{e^{-i\lambda} - e^{-ir\lambda}}{(1-e^{-i\lambda})^2} - \frac{(r-1)e^{-ir\lambda}}{1-e^{-i\lambda}} \right]. \qquad \text{(A.11)}$$

Using the analogue expression for $\sum_{k=1}^{r-1} k\, e^{-i(r-k)\lambda}$ and substituting $e^{i\lambda k} = \cos\lambda k + i \sin\lambda k$, it is straightforward to check that

$$2\pi\, f(\lambda) = \frac{1}{1+(r-1)\rho} \left[1 - \rho + \rho r - \frac{\rho}{r} \frac{1-\cos(\lambda r)}{r-\cos\lambda} \right] . \qquad \text{(A.12)}$$

Secondly, we consider $\lambda = 0$. Since $\cos(k\lambda) = 1$, $\sum_{k=1}^{r-1} k \cos(k\lambda) = \sum_{k=1}^{r-1} k = \frac{1}{2} r(r-1)$ proves the second equality in (6).

Proof of (25)

First, we prove that the lower right elements of $(\Sigma^{MA})^{-1}$ equal the lower right elements of B. It is readily verified that $\Sigma^{MA} = \sigma_e^2 CC'$ and $B = \sigma_e^{-2} DD'$ with

$$C = \begin{bmatrix} \cdot & & & \vdots \\ & \cdot & \vartheta_1 & \vartheta_2 \\ 0 & \cdot & 1 & \vartheta_1 \\ & & & 1 \end{bmatrix} \text{ and } D = \begin{bmatrix} \cdot & \cdot & & & \\ & \cdot & & & 0 \\ & \cdot & \psi_1 & \psi_0 & \\ \cdot & \cdot & \psi_2 & \psi_1 & \psi_0 \end{bmatrix}. \tag{A.13}$$

A sufficient condition for $(\Sigma^{MA})^{-1}B = I$ is then that $C'D = I$. Elaboration of this equality yields exactly the same conditions as $\vartheta(L)\psi(L) = 1$. To prove (25), use

$$A^{-1} = (\Sigma^{MA} + P)^{-1} = B\,(I + PB)^{-1} \text{ where } P = \begin{bmatrix} 0 & 0 \\ 0 & P_{22} \end{bmatrix},$$

and standard results on partitioned matrices yield the expression within curved brackets in the righthand side of (25).

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